PROBLEMAS DE AEROELASTICIDAD

Ejercicio 24

Se desea estudiar la respuesta de un avión a una ráfaga cosenoidal de la forma

$$
\begin{aligned}
W_G(s) &= \frac{W_o}{2}\left(1-\cos\left(\frac{2\pi s}{H}\right)\right) & \qquad 0 \le s \le H \\
&= 0 & \qquad s > H
\end{aligned}
$$

donde s es la variable tiempo adimensional $s = V_\infty t/b$, H el gradiente adimensional de la ráfaga, $H = 50$ y W_o la intensidad de la ráfaga. Para ello el avión se representa por un ala bidimensional libre de desplazarse verticalmente y las funciones de Kussner y de Wagner se aproximan por

$$\Psi(s) = 1 - \frac{1}{2}e^{-0.130s} - \frac{1}{2}e^{-s}$$

$$\Phi(s) = 1 - 0.165\, e^{-0.0455s} - 0.335\, e^{-0.300s}$$

Sea λ el parámetro másico definido como $\lambda = m/4\pi\rho b^2$.

Si para los casos de $\lambda = 5$ y de $\lambda = 50$ las raíces de la ecuación cubica

$p^3 + a_1p^2 + a_2p + a_3 = (p + \lambda_1)(p + \lambda_2)(p + \lambda_3)$ son

$\lambda = 5$ $\qquad \lambda_1 = -0.2545$ $\qquad \lambda_2 = -0.0693 + i\, 0.0\overset{181}{\cancel{174}}$

$\qquad\qquad \lambda_3 = -0.0693 + i\, 0.0\overset{181}{\cancel{174}}$

$\lambda = 50$ $\qquad \lambda_1 = -0.2966$ $\qquad \lambda_2 = -0.0433$ $\qquad \lambda_3 = -0.0106$

Se pide:

1) Obtenga la expresión de la respuesta en el dominio del Laplace.
2) Determinar el factor de carga definido como

$$\frac{\left(\dfrac{\ddot{\xi}(s)}{V_\infty W_o}\right)\max}{2\lambda b}$$

si el valor s en el que se alcanza el máximo es para $\lambda = 5$ $s = 3.8$ y para $\lambda = 50$ $s = 49$.

3) Comparar el resultado con la solución aproximada dada por la expresión $0.88\lambda/5.3 + l$.
4) Interprete físicamente los resultados obtenidos.

<u>Solución al problema #24</u>

$$M\ddot{\xi} = L_G + L_M$$

$$M\frac{U_\infty^2}{b^2}\frac{d^2\xi}{ds^2} = L_G(s) + L_M(s)$$

$$L_M(s) = -\pi\rho_\infty U_\infty^2\left[\frac{d^2\xi}{ds^2} + 2\int_0^s \phi(s-\sigma)\frac{d^2\xi}{d\sigma^2}\,d\sigma\right]$$

$$L_G = 2\pi\rho_\infty U_\infty^2\, b\int_0^s \psi'(s-\sigma)\frac{W_G(\sigma)}{U_\infty}\,d\sigma$$

Tomando la transformada de Laplace

$$M\frac{U_\infty^2}{b^2}\,p^2\bar{\xi} + \pi\rho_\infty U_\infty^2\left[p^2\bar{\xi} + 2\,\bar{\phi}\,p^2\bar{\xi}\right] =$$

$$2\pi\rho_\infty U_\infty b\;\bar{\Psi}\cdot p\bar{W}_G$$

$$p^2\bar{\xi} = \frac{2\pi\rho_\infty \cancel{U_\infty} b\,\bar{\Psi}\,p\,\bar{W}_G/U_\infty}{M\frac{\cancel{U_\infty^2}}{b^2} + \pi\rho_\infty\cancel{U_\infty^2}(1+2\bar{\phi})}$$

Dividiendo por $4\pi\rho_\infty$ obtenemos

$$\boxed{p^2\bar{\xi} = \frac{b/2\;\bar{\Psi}\,p\,\bar{W}_G/U_\infty}{\lambda + \frac{1}{4} + \frac{1}{2}\bar{\phi}}}$$

Tomando las aproximaciones

$$\psi(s) = 1 - \frac{1}{2}e^{-0.130s} - \frac{1}{2}e^{-s}$$

$$\phi(s) = 1 - 0.165e^{-0.0455s} - 0.335e^{-0.3s}$$

obtenemos la TxF de Laplace

$$\bar{\psi}(p) = \frac{1}{p} - \frac{1/2}{p+0.130} - \frac{1/2}{p+1}$$

$$\bar{\phi}(p) = \frac{1}{p} - \frac{0.165}{p+0.0455} - \frac{0.335}{p+0.3}$$

Tenemos que calcular ahora la Tx de Laplace de W_G

$$W_G(s) = \begin{cases} \frac{W_0}{2}\left(1 - \cos\left(\frac{2\pi s}{H}\right)\right) & 0 \leq s \leq H \\ 0 & s > H \end{cases}$$

$$\bar{W}_G(p) = \int_0^\infty e^{-ps} W_G(s)\,ds = \int_0^H e^{-ps}\frac{W_0}{2}\left(1 - \cos\left(\frac{2\pi s}{H}\right)\right)ds$$

$$= \frac{W_0}{2}\left[\frac{e^{-ps}}{-p}\Big|_0^H - \frac{e^{-ps}}{p^2+\left(\frac{2\pi}{H}\right)^2}\left(-p\cos\left(\frac{2\pi s}{H}\right) - \frac{2\pi}{H}\operatorname{sen}\left(\frac{2\pi s}{H}\right)\right)\Big|_0^H\right]$$

$$= \frac{W_0}{2}\left[\frac{1}{p} - \frac{e^{-pH}}{p} - \frac{e^{-pH}}{p^2+\left(\frac{2\pi}{H}\right)^2}\left(-p\cos 2\pi - \frac{2\pi}{H}\underbrace{\operatorname{sen}2\pi}_{0}\right)\right.$$

$$\left. + \frac{e^{-p\cdot 0}}{p^2+\left(\frac{2\pi}{H}\right)^2}\cdot(-p)\right]$$

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$$\bar{W}_G(p) = \frac{W_0}{2}\left[\frac{1}{p}(1-e^{-pH}) - \frac{p}{p^2+\left(\frac{2\pi}{H}\right)^2}(1-e^{-pH})\right]^{3/8}$$

$$\boxed{\bar{W}_G(p) = \frac{W_0}{2p}(1-e^{-pH})\frac{(2\pi/H)^2}{p^2+\left(\frac{2\pi}{H}\right)^2}}$$

Sustituyendo obtenemos

$$p^2\bar{\xi} = \frac{\frac{bW_0}{4U_\infty}\left(\frac{2\pi}{H}\right)^2 \frac{(1-e^{-pH})}{p^2+\left(\frac{2\pi}{H}\right)^2}\left(\frac{1}{p} - \frac{1/2}{p+0{,}130} - \frac{1/2}{p+1}\right)}{\lambda + \frac{1}{4} + \frac{1}{2}\left(\frac{1}{p} - \frac{0.165}{p+0.0455} - \frac{0.335}{p+0.3}\right)}$$

Reordenando términos obtenemos

$$\boxed{p^2\bar{\xi} = \frac{bW_0}{U_\infty}\left(\frac{\pi}{H}\right)^2 \frac{0.1412}{\lambda+0.250}\,\frac{p^3+0.5756p^2+0.09315p+0.003141}{(p+0.13)(p+1)(p^3+a_1p^2+a_2p+a_3)}\,\frac{1}{p^2+\left(\frac{2\pi}{H}\right)^2}(1-e^{-pH})}$$

Llamando $\mathcal{N}(p) = p^3+0.5756p^2+0.09315p+0.003141$

$$\mathcal{D}(p) = (p+0.13)(p+1)(p^3+a_1p^2+a_2p+a_3)\left(p^2+\left(\frac{2\pi}{H}\right)^2\right)$$

con $a_1 = \dfrac{0.3455\lambda+0.3364}{\lambda+0.25}$; $a_2 = \dfrac{0.01365\lambda+0.1438}{\lambda+0.250}$

$$a_3 = \frac{0.006825}{\lambda+0.25}$$

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$$\ddot{\xi}(s) = \frac{bW_0}{U_\infty}\left(\frac{\pi}{H}\right)^2 \frac{0.1412}{\lambda+0.25}\,\mathcal{L}^{-1}\left[\frac{\mathcal{N}(p)}{\mathcal{D}(p)}\left(1-e^{-pH}\right)\right]$$

Sea $f(s) = \mathcal{L}^{-1}\dfrac{\mathcal{N}(p)}{\mathcal{D}(p)}$ por el teorema del retardo de la transformada de Laplace

$$\mathcal{L}^{-1}\left[\frac{\mathcal{N}(p)}{\mathcal{D}(p)}\left(1-e^{-pH}\right)\right] = f(s) - f(s-H).$$

donde $f(s-H)$ solo esta definida para $s>H$.

$$\boxed{\ddot{\xi}(s) = \frac{bW_0}{U_\infty}\left(\frac{\pi}{H}\right)^2 \frac{0.1412}{\lambda+0.25}\left[f(s) - f(s-H)\right]}$$

Pero

$$f(s) = A_1 e^{-0.130s} + A_2 e^{-s} + B_1 e^{-\gamma_1 s} + B_2 e^{-\gamma_2 s} + B_3 e^{-\gamma_3 s} + B_4 e^{-i\frac{2\pi}{H}s} + B_5 e^{i\frac{2\pi}{H}s}$$

Donde γ_1, γ_2 y γ_3 son las raíces del polinomio cúbico

$A_1, A_2, B_1, B_2, B_3, B_4$ y B_5 son los residuos del cociente $\dfrac{\mathcal{N}(p)}{\mathcal{D}(p)}$

$\underline{\underline{\lambda = 5}}$ $\quad a_1 = 0.3931238;\ a_2 = 0.0403905;\ a_3 = 0.013000$

$$\frac{\dfrac{1}{p} - \dfrac{1/2}{p+0.13} - \dfrac{1/2}{p+1}}{\lambda + \dfrac{1}{4} + \dfrac{1}{2}\left(\dfrac{1}{p} - \dfrac{0.165}{p+0.0455} - \dfrac{0.335}{p+0.3}\right)} =$$

$$\frac{0.1412}{\lambda + \frac{1}{4}} \cdot \frac{p^3 + 0.5756p^2 + 0.09315p + 0.003141}{(p+0.13)(p+1)(p^3 + a_1p^2 + a_2p + a_3)}$$

$$\frac{\dfrac{(p+0.13)(p+1) - \frac{1}{2}p(p+1) - \frac{1}{2}(p)(p+0.13)}{\cancel{p}(p+0.13)(p+1)}}{\dfrac{p(p+0.0455)(p+0.3)(\lambda+\frac{1}{4}) + \frac{1}{2}\left((p+0.0455)(p+0.3) - 0.165p\cdot(p+0.3) - 0.335p(p+0.0455)\right)}{\cancel{p}(p+0.0455)(p+0.3)}}$$

$$\frac{(p+0.3)(p+0.0455)\left[(p+0.13)(p+1) - \frac{1}{2}p(p+1) - \frac{1}{2}p(p+0.13)\right]}{(p+1)(p+0.13)\left[p(p+0.0455)(p+0.3)(\lambda+\frac{1}{4}) + \frac{1}{2}\left((p+0.0455)(p+0.3) - 0.165p(p+0.3) - 0.335p(p+0.0455)\right)\right]}$$

$$\cancel{p^2} + 1{,}13p + 0.13 - \cancel{\frac{p^2}{2}} - 0.5p - \cancel{0.5p^2} - 0.065p$$

$$= 0.565p + 0.13$$

$$\boxed{N(p) = (p+0.3)(p+0.0455)(0.565p+0.13)}$$

Denominador

$$p(p+0.0455)(p+0.3) = p^3 + 0.3455p^2 + 0.0137p$$

$$\frac{1}{2}(p+0.0455)(p+0.3) = \frac{1}{2}p^2 + 0.1728p + 0.0069$$

$$-\frac{0.165}{2}p(p+0.3) = -0.0825p^2 - 0.0248p$$

$$-\frac{0.335}{2}p(p+0.0455) = -0.1675p^2 - 0.0076p$$

Sumando

$$(\lambda+\tfrac{1}{4})(p^3+0.3455p^2+0.0137p) + 0.25p^2 + 0.1404p + 0.0069$$

$$(\lambda+\tfrac{1}{4})\left[p^3 + \left(0.3455 + \frac{0.25}{\lambda+\frac{1}{4}}\right)p^2 + \left(0.0137 + \frac{0.1404}{\lambda+\frac{1}{4}}\right)p + \frac{0.0069}{\lambda+\frac{1}{4}}\right]$$

$$\boxed{\Phi(p) = (p+1)(p+0.13)\left(p^2+\left(\frac{2\pi}{H}\right)^2\right)\left[p^3+a_1p^2+a_2p+a_3\right]}$$

$\lambda = 5$ 7/8

$\gamma_1 = -0.2546, \quad \gamma_2 = -0.0693 + 0.0181 i$

$\gamma_2 = -0.0693 - 0.0181 i$

$B_1 = -0.0004$

$B_2 = 0.0592 + 0.0747 i$

$B_3 = 0.0592 - 0.0747 i$

$A_2 = -0.0004$

$A_1 = -0.0573$

$B_5 = -0.0302 - 0.0165 i$

$B_4 = -0.0302 + 0.0165 i$

$$\cancel{\left(\frac{\ddot{\xi}}{\frac{bW_0}{U_\infty}}\right)_{max}} \qquad \frac{d^2\xi}{ds^2} = \frac{b^2}{U_\infty^2}\frac{d^2\xi}{dt^2}$$

$$s = \frac{U_\infty t}{b}$$

$$\frac{d^2\xi}{dt^2} = \frac{U_\infty^2}{b^2}\frac{d^2\xi}{ds^2} = \frac{W_0 U_\infty}{b}\left(\frac{\pi}{H}\right)^2 \frac{0.1412}{\lambda + 0.25}\left[f(s) - f(s-H)\right]$$

$$\frac{\frac{d^2\xi}{dt^2}}{\frac{W_0 U_\infty}{2\lambda b}} = \left(\frac{\pi}{H}\right)^2 \frac{0.2824\lambda}{\lambda + 0.25}\left[f(s) - f(s-H)\right]$$

Para $\lambda = 5$ $\left(\frac{\pi}{H}\right)^2 \frac{0.2824\lambda}{\lambda + 0.25} = \left(\frac{\pi}{H}\right)^2 0.26895 = 0.00106$

$\left(\frac{\pi}{H}\right)^2 = 0.00395$

$\cancel{\ddot{\xi} \le 0.0711}$

$$\boxed{\left(\frac{\frac{d^2\xi}{dt^2}}{\frac{W_0 U_\infty}{2\lambda b}}\right)_{max} = 0.4221}$$

$d_n = 0.0711$

$\lambda = 50$

$\gamma_1 = -0.2966 \quad \gamma_2 = -0.0433 \quad \gamma_3 = -0.0106$

$B_1 = 0$

$B_2 = -0.0007$

$B_3 = 0.0111$

$A_2 = 0$

$A_1 = -0.0017$

$B_5 = -0{,}0043 + 0.0010\,i$

$B_4 = -0{,}0043 - 0.0010\,i$

$d_n = 0.0162$ máximo ocurre en $\underline{s = 37}$

7 vale

$$\boxed{\frac{\ddot{\xi}}{\frac{U_\infty W_0}{2\lambda b}} = 0{,}8126}$$